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APPLICATION OF SPEED IMPROVEMENT TECHNIQUES IN COORDINATE TRANS--ETC(U)

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Application of Speed Improvement Techniques in Coordinate Transformations for the Multiple Platform Sensor Integration Problem

B.H. CANTRELL

Radar Analysis Staff
Radar Division

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NAVAL RESEARCH LABORATORY
Washington, D.C.

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A means of improving the speed of computation in performing coordinate transformation used in multiple platform sensor integration problems is described. The method is based on quaternions.

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INTRODUCTION

In multiple platform sensor integration a large number of coordinate transformations are required. Therefore it is necessary to improve the speed of calculations as much as possible. There are at least two fundamentally different starting points in which coordinate rotations can be derived. The first is the use of direction cosine matrices which are well known, published, and used repeatedly in the literature. The second means is to derive the coordinate rotations by using quaternions. This method may not be as well known. Rather than develop the quaternions an example is first shown using the direction cosine and then the quaternion approach. Since both methods yield the same results for the same initial conditions they are equivalent. In fact identities relate the two methods. It is the purpose of this report to describe to those members of the sensor integration community (who may not be aware of it) how these identities can be used to improve the computation time in coordinate rotations.

EXAMPLE

A simple rotation of the X,Y coordinates into the X', Y' coordinates by the angle α is shown in Fig. 1.

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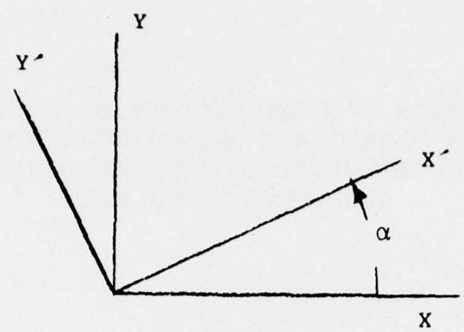

Fig. 1 — Demonstration of coordinate rotation using direction cosines

The equations are well known and are as follows:

$$X' = X \cos \alpha + Y \sin \alpha \tag{1}$$

$$Y' = -X \sin \alpha + Y \cos \alpha \quad . \tag{2}$$

The same transformation in quaternions is

$$X' = \frac{2\,(X + Y \tan \alpha/2)}{1 + \tan^2 \alpha/2} - X \tag{3}$$

$$Y' = Y - \frac{2(X+Y \tan \alpha/2)}{1 + \tan^2 \alpha/2} \tan \alpha/2 \quad . \tag{4}$$

We will shown (1) and (2) are equivalent to (3) and (4). Expanding (3) and (4) one obtains

$$X' = \frac{(1 - \tan^2 \alpha/2)}{(1 + \tan^2 \alpha/2)} X + \frac{2 \tan \alpha/2}{1 + \tan^2 \alpha/2} Y \quad (5)$$

$$Y' = \frac{-2 \tan \alpha/2}{1 + \tan^2 \alpha/2} X + \frac{(1 - \tan^2 \alpha/2)}{1 + \tan^2 \alpha/2} Y. \quad (6)$$

By using the trigometric identities

$$\cos \alpha = \frac{1 - \tan^2 {}^{\alpha}/2}{1 + \tan^2 {}^{\alpha}/2} \quad \text{and} \quad (7)$$

$$\sin \alpha = \frac{2 \tan {}^{\alpha}/2}{1 + \tan^2 {}^{\alpha}/2}, \quad (8)$$

we find (1) and (2) are equivalent to (3) and (4).

THE METHOD

In the example used the quaternion technique would be faster to compute because only one trigometric function need be computed. In addition, the tangent of the half angle is also an easier computation as discussed subsequently. The object then is to compute the tangent of the half angle and through the use of trigometric identities to rearrange the direction cosine equations so that the least computation is obtained.

Consider the following sequence of computations:

$$b = \tan \alpha/2 \quad (9)$$

$$D = 2/(1 + b^2) \quad (10)$$

$$X' = D(X + bY) - X \tag{11}$$

$$Y' = Y - bD(X+bY) \tag{12}$$

for example. Equation (11) and (12) are equivalent to either (1) and (2) or (3) and (4) when (9) and (10) are substituted into (11) and (12). The calculation requires the evaluation of one trigometric function, 5 multiplies or divides, and 4 additions or subtractions. More complex rotations can be improved in the computation time by applying similar principles. The principal is to modify the sine and cosine functions by trigometric identities so that they are in terms of the tangent of the half angle and then rearrange the direction cosine equations for the least computation.

A few comments are made on the evaluation of the tangent. Since the range of values of α is from -180° to 180° the range of values for the half angle $\alpha/2$ is -90° + 90°. However only the values from 0° to 90° need be computed because of the odd symmetry. This range of values can be further reduced by using the trigometric identity.

$$\tan \alpha/2 = 1/[\tan (90° - \alpha/2)] \tag{13}$$

for values of $\alpha/2$ between 45° and 90°. Therefore, the only values required for computation are those of $\alpha/2$ between 0° and 45°. The computation can be made very accurate with the table lookup and a linear interpolation between points.

SUMMARY

Coordinate rotations can be derived in two fundamentally different ways. The quaternion representation has a speed advantage to the direction cosine representation. It was the purpose of this report to point out to those in the sensor integration community this fact. The basic principle which can be applied is to find the tangent of the half angle and rearrange the direction cosine equations using trigometric identities to obtain the higher speed forms. The technique is applicable to either digital computers or high speed dedicated digital hardware where extremely high speeds are desired.